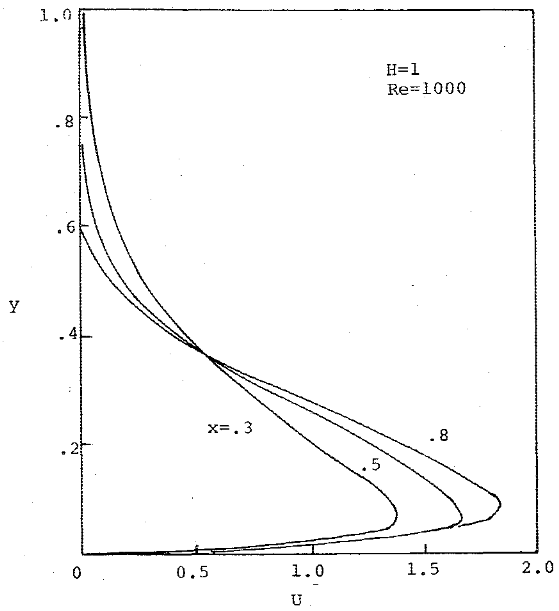

Fig. 2 Velocity distribution in the wall jet.

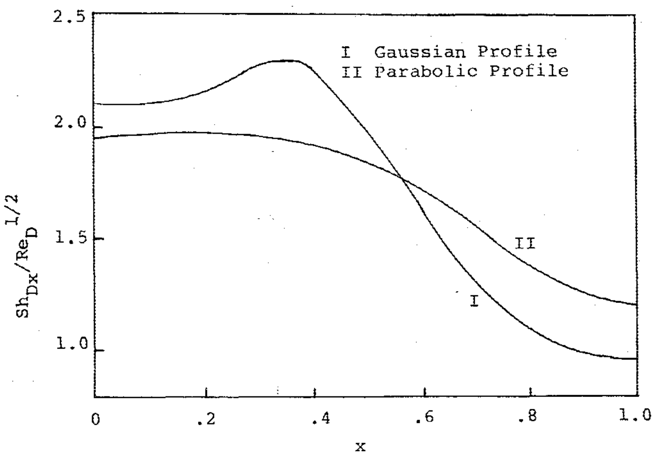

Fig. 3 Variation of local Sherwood number ($Sc = 2.45$).

with the model of Sparrow[4] which assumed negligible diffusion of vorticity in the direction normal to the axis of the primary jet flow. The velocity profiles in the wall jet are shown in Fig. 2, the distance from the deflecting wall to the point of maximum velocity increases with distance from the stagnation point. However, this distance from the wall does not correspond to a boundary-layer thickness as assumed by the Scholtz model.[2] The U-velocity profile was found to be invariant for $x \geq 0.25$; a freestream velocity in the sense discussed by Scholtz and Trass was not confirmed. This observation was independent of the initial jet velocity profile examined in this study. The profiles in Fig. 2 agree well with those obtained experimentally by Donaldson et al.[6]

The results for the heat and mass transfer computation are shown in Figs. 3 and 4, and the corresponding initial velocity profiles are shown inset in Fig. 4. In the mass transfer study, the property of Naphthalene is assumed, $Sc = 2.45$. The parabolic initial profile predicts Sherwood number within 2% of the experimental data of Sparrow.[4] The Gaussian profile overpredicted data by 9%. The Nusselt number obtained in this study is in good agreement with the values obtained for the stagnation region of a cylinder in crossflow.[9]

Conclusions

The variation of heat and mass transfer parameters along a deflecting surface is controlled by the initial velocity profile in a close-proximity impinging jet. The difference between the transfer parameters obtained for the Gaussian and parabolic velocity profiles may be explained by the quantity of high-velocity fluid delivered to the deflecting wall. For the same mean flow velocity based on jet width, the Gaussian profile attains a higher centerline velocity. The effect of the entrainment of ambient fluid did not seem to penetrate the wall jet.

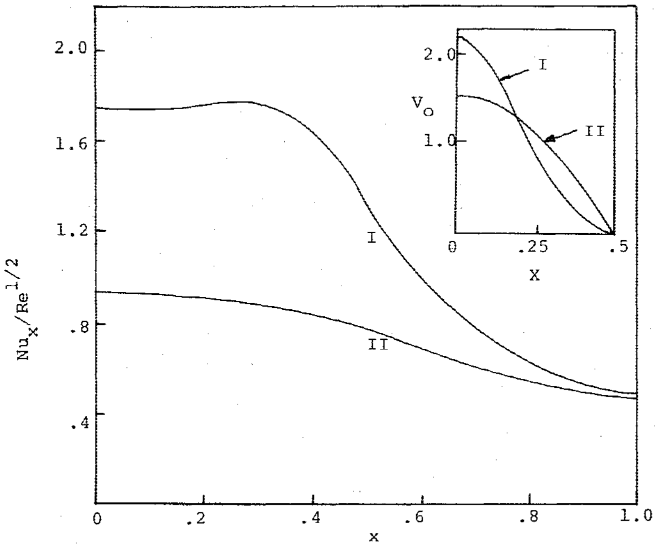

Fig. 4 Local heat-transfer coefficient ($Pr = 0.72$, $H = 1$).

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Notes on the Transonic Indicial Method

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Introduction

RECENTLY, Ballhuas and Goorjian[1] presented a method of calculating flutter derivatives for two-dimensional

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airfoils at transonic speeds, using the indicial method previously developed[2] for incompressible flow. The indicial method uses the principle of superposition to relate complex motions to an indicial response by means of Duhamels integral. The indicial response is the time history of the derivative in question in response to a step change in some flow parameter. In the work of Ballhaus and Goorjian,[1] the indicial response for transonic flows is calculated using the nonlinear ADI scheme[3] of the same authors; only calculations of the lift and pitching moment coefficients are included, and no explicit attempt is made to treat the shock motion. Because the principle of superposition is used, it is assumed that the unsteady flow can be regarded as a linear perturbation to the mean steady flow. Although this is a satisfactory assumption for most of the flowfield, it is not valid for the pressure distribution in the region bounded by the extremities of the shock motion, since the pressure there can jump from a postshock value to a preshock value as the shock traverses the region. The allied problem of a steady perturbation in which the shock moves has been discussed by Nixon,[4] and a method for treating the difficulty associated with a moving shock, using the technique of strained coordinates, is presented. It is the purpose of this Note to investigate the effect of the shock motion on the flow variables in the indicial method.

The analysis shows that the shock motion must be explicitly included if the pressure coefficient is to be calculated. A formula for the pressure coefficient is derived with proper treatment of the shock motion, thus allowing the computation of the pressure distribution for general oscillations using the indicial method. However, if only the lift and pitching moment coefficients are to be calculated, then no explicit treatment of the shock motion is necessary; the shock motion is implicitly taken into account in the computation of the indicial response.

The Indicial Method

Consider the change in the pressure coefficient $C_p(x,z,t)$ due to an infinitesimal change in some parameter $\epsilon(\tau)$ at some time τ. If the pressure distribution varies continuously with ϵ, then a Taylor's series expansion gives

$$\Delta C_p(x,z,t) = C_{p_\epsilon}(x,z,t,\tau)\frac{d\epsilon(\tau)}{d\tau}\Delta\tau + \begin{pmatrix}\text{higher order}\\ \text{terms}\end{pmatrix} \tag{1}$$

where t is the time, $C_{p_\epsilon}(x,z,t,\tau)$ the rate of change of $C_p(x,z,t)$ with ϵ at some time τ and $\Delta C_p(x,z,t)$ the change in pressure distribution. Neglecting the higher order terms, the total effect of such steps up until time t is then

$$\Delta C_p(x,z,t) = \epsilon(0)C_{p_\epsilon}(x,z,t,0) + \int_0^t C_{p_\epsilon}(x,z,t,\tau)\frac{d\epsilon(\tau)}{d\tau}d\tau \tag{2}$$

If it can be assumed that the behavior of C_p with ϵ is linear, then $C_{p_\epsilon}(x,z,t,\tau)$ can be represented by its value at $\tau=0$, provided that time t is taken relative to τ. Thus,

$$C_{p_\epsilon}(x,z,t,\tau) = C_{p_\epsilon}(x,z,t-\tau,0) \tag{3}$$

Equation (2) then becomes

$$\Delta C_p(x,z,t) = \epsilon(0)C_{p_\epsilon}(x,z,t,0) + \int_0^t C_{p_\epsilon}(x,z,t-\tau,0)\frac{d\epsilon(\tau)}{d\tau}d\tau \tag{4}$$

By a simple change of variable, Eq. (4) can then be written as

$$\Delta C_p(x,z,t) = \epsilon(0)C_{p_\epsilon}(x,z,t) + \int_0^t C_{p_\epsilon}(x,z,\tau)\frac{d\epsilon(t-\tau)}{d\tau}d\tau \tag{5}$$

where the functional form of $C_{p_\epsilon}(x,z,t,0)$ has been contracted to $C_{p_\epsilon}(x,z,t)$ for convenience in presentation. Equation (5) is the form of a typical relation in the indicial method. It is assumed that $C_{p_\epsilon}(x,z,t)$ can be obtained by some means. A more detailed description of the indicial method is given by Tobak.[2]

Shock Motion Analysis

In the foregoing analysis it has been assumed that the variation of $C_p(x,z,t)$ with ϵ is linear. For continuous flows governed by a nonlinear equation, this assumption may well be adequate for small values of ϵ. However, as pointed out in Ref. 4, if there are shock waves that move during the motion, the assumption of linear variation is totally inadequate in the region bounded by the extremities of the shock motion. This is because the pressures in this region can jump from a preshock value to a postshock value or vice versa as the shock traverses this region. It is pointed out in Ref. 4 that this difficulty can be overcome by the use of a strained coordinate system in which the shock remains at the same location.

Let the strained coordinate system be given by (x',z',t), which is related to the physical coordinate system (x,z,t) by

$$x = x' + \delta x_s(t)x_1(x'), \qquad z = z' \tag{6}$$

where $\delta x_s(t)$ is the change in shock location at some time t, and $x_1(x')$ is the straining function used in Ref. 4, namely,

$$x_1(x') = \frac{x'(1-x')}{x_s'(1-x_s')} \quad 0 \le x' \le 1$$
$$x_1(x') = 0 \quad x'<0, \quad x'>1 \tag{7}$$

where x_s' is the location of the shock wave in the strained coordinate system. In Eqs. (6) and (7) it is assumed that the shock waves are normal to the freestream.

For low reduced frequencies, the pressure coefficient, $C_p(x,z,t)$, is given[3] by

$$C_p(x,z,t) = -(2\beta^2/k)u(x,z,t) \tag{8}$$

where $u(x,z,t)$ is the scaled perturbation velocity in the x direction and, following the ideas of Ref. 4, is given by

$$u(x,z,t) = u_0(x',z)[1-\delta x_s(t)x_{1_{x'}}(x')] + u_1(x',z',t) \tag{9}$$

where

$$k = (\gamma+1)M_\infty^q, \qquad \beta = (1-M_\infty^2)^{1/2} \tag{10}$$

and $u_0(x',z')$ is the value for the mean steady state. The exponent q is the usual transonic exponent in the transonic small-disturbance equation. The coordinates (x',z') and (x,z) are related by Eqs. (6) and (7).

In Eq. (9), $\delta x_s(t)$, $u_1(x',z',t)$ depend linearly on the parameter $\epsilon(\tau)$ and can therefore be treated by the indicial method. The nonlinear effect appears implicitly through the transformation from the strained coordinates to the physical coordinates. Thus,

$$\delta x_s(t) = \delta x_{s_\epsilon}(t)\epsilon(0) + \int_0^t \delta x_{s_\epsilon}(\tau)\frac{d\epsilon(t-\tau)}{dt}d\tau \tag{11}$$

$$u_1(x',z',t) = u_{1_\epsilon}(x',z',t)\epsilon(0) + \int_0^t u_{1_\epsilon}(x',z',\tau)\frac{d\epsilon(t-\tau)}{d\tau}d\tau \tag{12}$$

where $\delta x_{s_\epsilon}(t)$ and $u_{1_\epsilon}(x',z',t)$ are the indicial responses of $\delta x_s(t)$ and $u_1(x',z,t)$, respectively.

The expression for the pressure coefficient can then be obtained from Eqs. (8, 9, 11, and 12). Thus,

$$C_p(x,z,t) = -\frac{2\beta^2}{k}\Big\{\Big[u_0(x',z') + u_{1_\epsilon}(x',z',t)\epsilon(0) + \int_0^t u_{1_\epsilon}(x',z',\tau)\frac{d\epsilon(t-\tau)}{d\tau}d\tau\Big] - \Big[u_0(x',z')x_{1_{x'}}(x')\Big(\delta x_{s_\epsilon}(t)\epsilon(0) + \int_0^t \delta x_{s_\epsilon}(\tau)\frac{d\epsilon(t-\tau)}{d\tau}d\tau\Big)\Big]\Big\} \tag{13}$$

where x and z are given by Eqs. (6) and (11). This relation is essentially the same as that given in Ref. 1, except for the last term in square brackets in Eq. (13) and the coordinate straining (x',z'), which are both functions of the shock increment. The indicial method can therefore, be used to compute pressure distributions through Eq. (13).

In the approach of Ballhaus and Goorjian,[1] no explicit account is taken of the shock wave motion, and only the lift coefficient C_L and pitching moment C_m are calculated. The effect of including the shock motion terms explicitly in the calculation of C_L and C_m is now deduced.

The lift coefficient C_L is defined by the relationship

$$C_L(t) = \int_0^1 [C_p(x,-0,t) - C_p(x,+0,t)]dx \tag{14}$$

For simplicity, consider only one component of Eq. (14), that is

$$I = \int_0^1 C_p(x,+0)dx$$

or

$$I = -\frac{2\beta^2}{k}\int_0^1 u(x,+0,t)dx \tag{15}$$

where $u(x,+0,t)$ is given by Eq. (9).

In Eq. (9) the term $u_1(x',z,t)$ is found from Eq. (12) and the indicial response characterized by the parameter ϵ_0. Thus,

$$u_{1_\epsilon}(x',z',t) = \frac{1}{\epsilon_0}\Big\{u^1(x^1,z,t) - u_0(x',z') \times [1-\delta x_{s0}(t)x_{1_{x'}}(x')]\Big\} \tag{16}$$

where $u^1(x^1,z,t)$ is the indicial response for $u(x,z,t)$ and

$$x^1 = x' + \delta x_{s0}(t)x_1(x') \tag{17}$$

and $\delta x_{s0}(t)$ is the indicial response for the movement of the shock location. Combination of Eqs. (9, 12, and 16) in Eq. (15) gives

$$I = -\frac{2\beta^2}{k}\Big\{\int_0^1 u_0(x',0)\Big[1-\delta x_s(t)x_{1_{x'}}(x')\Big]dx + \frac{1}{\epsilon_0}\Big\{\int_0^1\Big[\epsilon(0)u^1(x^1,+0,t) + \int_0^t u^1(x^1,+0,t)\frac{d\epsilon(t-\tau)}{dt}\Big]dx - \int_0^1\Big\{\epsilon(0)u_0(x',z')\Big[1-\delta x_{s0}(t)x_{1_{x'}}(x')\Big] + \int_0^t u_0(x',z')\Big[1-\delta x_{s0}(t)x_{1_{x'}}(x')\Big]\frac{d\epsilon(t-\tau)}{dt}d\tau\Big\}dx\Big\}\Big\} \tag{18}$$

From Eq. (6)

$$dx = [1+\delta x_s(t)x_{1_{x'}}(x')]dx' \tag{19a}$$

and from Eqs. (6) and (17)

$$dx = \{1+[\delta x_s(t) - \delta x_{s0}(t)]x_{1_{x'}}(x')\}dx^1 \tag{19b}$$

Substitution of Eqs. (19) into Eq. (18) then gives to first order in the parameter ϵ

$$I = -\frac{2\beta^2}{k}\Big[A_0 + \epsilon(0)\bar{A}_\epsilon(t) + \int_0^t \bar{A}_\epsilon(\tau)\frac{d\epsilon(t-\tau)}{dt}d\tau\Big] \tag{20}$$

where

$$A_0 = \int_0^1 u_0(x',+0)dx'$$

$$\bar{A}_\epsilon(t) = \frac{1}{\epsilon_0}\Big\{\Big[\int_0^1 u^1(x^1,+0,t)dx^1 - A_0\Big] + [\delta x_s(t) - \delta x_{s0}(t)][B_1 - B_0]\Big\} \tag{21}$$

and

$$B_1 = \int_0^1 u^1(x^1,+0,t)x_{1_{x^1}}(x^1)dx^1$$

$$B_0 = \int_0^1 u_0(x',+0)x_{1_{x'}}(x')dx' \tag{22}$$

In Eq. (21), both of the terms $[\delta x_s(t) - \delta x_{s0}(t)]$ and $[B_1 - B_0]$ are the same order of magnitude as the motion and, hence, to first-order the last term in square brackets in Eq. (21) can be neglected. Thus,

$$\bar{A}_\epsilon(t) = \frac{1}{\epsilon_0}\Big[\int_0^1 u^1(x^1,+0,t)dx^1 - A_0\Big] \tag{23}$$

If a similar procedure is applied to the lower surface term in Eq. (14), then it may be shown that

$$C_L(t) = \frac{\beta^2}{k}\Big[\Big(\frac{k}{\beta^2}\Big)^0 C_{L_0} + \epsilon(0)\bar{C}_{L_\epsilon}(t) + \int_0^t \bar{C}_{L_\epsilon}(\tau)\frac{d\epsilon(t-\tau)}{dt}d\tau\Big] \tag{24}$$

where C_{L0} is the value of the lift coefficient of the base solution and

$$\bar{C}_{L_\epsilon}(t) = \frac{1}{\epsilon_0}\Big[\Big(\frac{k}{\beta^2}\Big)^1 C_L^1(t) - C_{L_0}\Big(\frac{k}{\beta^2}\Big)^0\Big] \tag{25}$$

and $C_L^1(t)$ is the indical response of the lift coefficient. The superscripts "0" and "1" denote values for the base solution and the indicial response, respectively. If the Mach number is unchanged by the motion, then (k/β^2) is constant and hence

$$C_L(t) = C_{L_0} + \epsilon(0)C_{L_\epsilon}(t) + \int_0^t C_{L_\epsilon}(\tau)\frac{d\epsilon(t-\tau)}{dt}d\tau \tag{26}$$

where

$$C_{L_\epsilon}(t) = \frac{\beta^2}{k}\bar{C}_{L_\epsilon}(t) \quad (27)$$

A similar analysis for the pitching moment C_m results in

$$C_m(t) = C_{m_0} + \epsilon(0)C_{m_\epsilon}(t) + \int_0^t C_{m_\epsilon}(\tau)\frac{d\epsilon(t-\tau)}{dt}d\tau \quad (28)$$

where

$$C_{m_\epsilon}(t) = 1/\epsilon_0[C_m^I(t) - C_{m_0}] \quad (29)$$

and $C_m^I(t)$ is the indicial response for the pitching moment.

Equations (26) and (28) are the same equations used by Ballhaus and Goorjian.[1] Hence, while the shock motion must be taken into account explicitly when the pressure coefficient is to be calculated, it need not be for the calculation of integrated parameters such as C_L and C_m. The reason for this is that the shock movement is implicitly taken into account in the computation of the indicial responses $C_{L_\epsilon}(\epsilon)$ and $C_{m_\epsilon}(t)$.

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Nonlinear Formulation for Low-Frequency Transonic Flow

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Introduction

WE will consider the two-dimensional unsteady transonic flow past a thin airfoil executing small-amplitude harmonic oscillations. The usual approach[1] solves a (time) Fourier-transformed problem linearized about a prescribed steady flow. The mean flow is obtained (once) from a Murman-Cole method, and its solution determines the variable coefficients of a linearized, frequency-dependent, mixed-type problem. This linearization, however, is not uniformly valid in time: the *nonharmonic* part of the total flow *must* change for different disturbance frequencies and amplitudes because the problem is nonlinear. The mean flow cannot be assumed known for all time; the "ϵ expansion" (i.e., the straightforward linearization) therefore breaks down because it does not describe the "back-interaction" mechanism that arises from the nonlinear harmonic interplay required on physical grounds.

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This defect is remedied by equating *not* coefficients of like powers of a small-amplitude measure (in determining a sequential ordering of problems), but coefficients of like *harmonics*. The resulting equations describe the feedback phenomenon, and if they can be solved exactly, no physical approximation will be involved. However, for simplicity, we will consider the effect of the primary harmonic only and use the small-disturbance formulation (there is no conceptual difficulty in writing down more terms, however). The back-interaction effect, of course, will be important for low-frequency problems where shock excursions are generally large, and may have a significant effect on mean shock jumps and location. For high frequencies, the effect is probably less important, and a linearized approach should do. This Note shows how flutter criteria depend on both frequency and amplitude boundaries in a physically self-consistent way.

Analysis

Let M_∞ be the subsonic freestream Mach number, U the freestream speed, C the chord, δ the thickness ratio, and γ the ratio of specific heats. We normalize the streamwise coordinate x_0, the normal coordinate y_0, and the time t_0 by defining nondimensional variables by $x = x_0/C$, $y = [(1+\gamma)\delta M_\infty^2]^{1/3} y_0/C$, and $t = [(1+\gamma)\delta M_\infty^2]^{1/3} Ut_0/M_\infty^2 C$, and introduce a nondimensional disturbance potential φ by expanding the total dimensional potential in the form

$$\Phi \cong UCx + \frac{\delta^{2/3}UC}{[(1+\gamma)M_\infty^2]^{1/3}}\varphi(x,y,t)$$

Let us also introduce a reduced frequency $k = \omega C/U$, where ω is the frequency of the oscillation, and a nondimensional frequency $\Omega = M_\infty^2 k/[(1+\gamma)\delta M_\infty^2]^{1/3}$. Substitution into the full potential equation leads to the small-perturbation equation

$$(K^* - \varphi_x)\varphi_{xx} + \varphi_{yy} = 2\varphi_{xt} + (k/\Omega)\varphi_{tt} \quad (1)$$

where the transonic similarity parameter $K^* = (1-M_\infty^2)/[(1+\gamma)\delta M_\infty^2]^{2/3}$. Let $y = g_{u,\ell}(x,t)$ represent upper and lower wing surfaces. Then, Eq. (1) is solved along with the following linearized boundary conditions:

$$\varphi_y(0<x<1;\quad y=\pm 0) = \left(\frac{\partial}{\partial x} + \frac{K}{\Omega}\frac{\partial}{\partial t}\right)g_{u,\ell}(x,t) \quad (2)$$

$$\left(\varphi_x + \frac{k}{\Omega}\varphi_t\right)_{\substack{x>1\\y=0+}} = \left(\varphi_x + \frac{k}{\Omega}\varphi_t\right)_{\substack{x>1\\y=0-}} \quad (3)$$

$$\varphi_x^2 + \varphi_y^2 \to 0 \quad \text{as } x^2 + y^2 \to \infty \quad (4)$$

We assume that the unsteady motion is a small perturbation on the steady flow, and characterize it by a small nondimensional displacement ϵ and the reduced frequency. In view of the preceding discussion, we expand both the airfoil motion and the disturbance potential in *real* harmonic series,

$$g(x,t) = g_0(x) + \tfrac{1}{2}\epsilon\{g_1(x)e^{i\Omega t} + \bar{g}_1 e^{-i\Omega t}\} + \ldots \quad (5)$$

$$\varphi(x,y,t) = \varphi_0(x,y) + \tfrac{1}{2}\epsilon\{\varphi_1(x,y)e^{i\Omega t} + \bar{\varphi}_1 e^{-i\Omega t}\} + \ldots \quad (6)$$

bars denoting complex conjugates (higher harmonics are related to higher-order amplitude effects that can be consistently neglected). Substitution in Eqs. (1-3) leads to the mean flow formulation

$$\frac{\partial}{\partial x}\left\{K^*\varphi_{0_x} - \frac{1}{2}\varphi_{0_x}^2 - \frac{1}{4}\epsilon^2|\varphi_{1_x}|^2\right\} + \frac{\partial}{\partial y}\{\varphi_{0_y}\} = 0 \quad (7)$$

$$\varphi_{0_y}(0<x<1;\quad y=\pm 0) = \partial g_{0_{u,\ell}}(x)/\partial x \quad (8)$$